



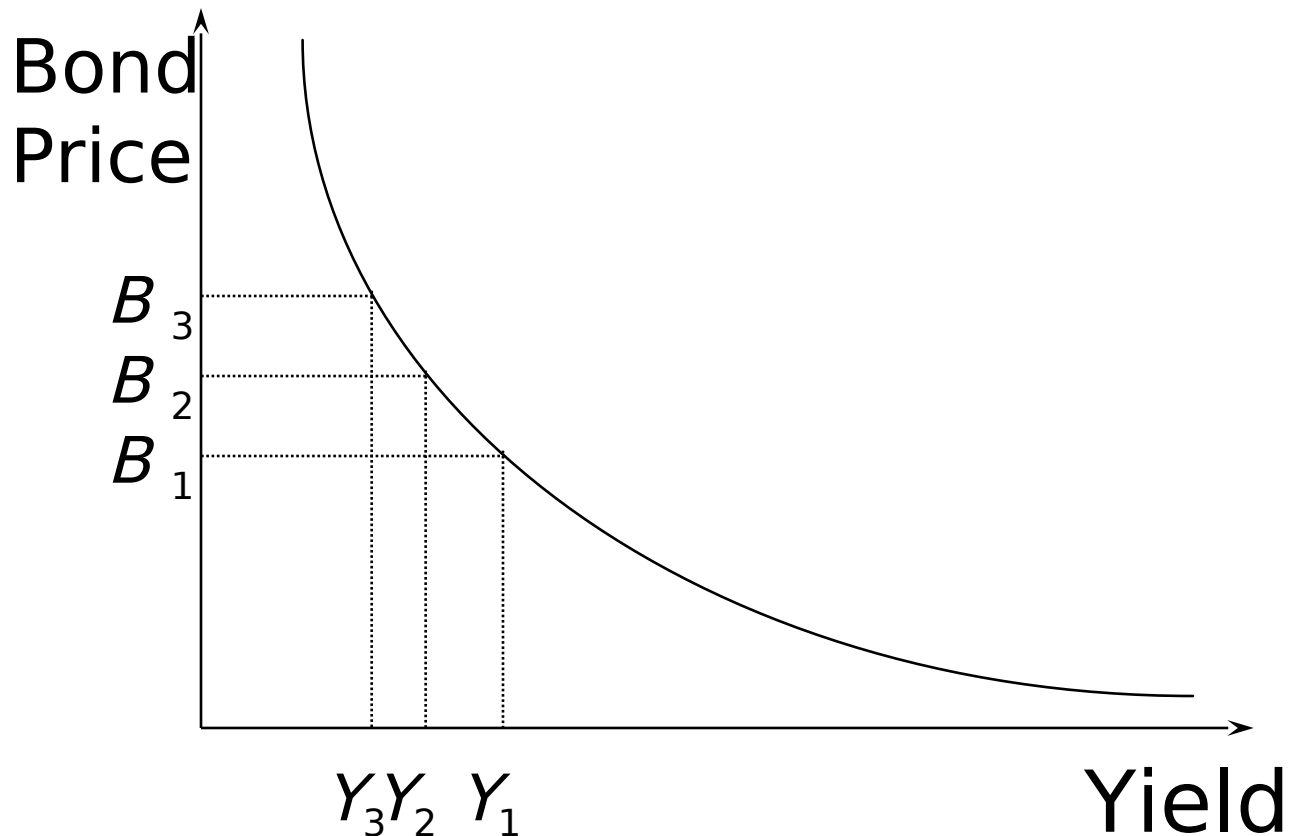
Quanto, Timing, and Convexity Adjustments

Chapter 29

Forward Yields and Forward Prices

- We define the forward yield on a bond as the yield calculated from the forward bond price
- There is a non-linear relation between bond yields and bond prices
- It follows that when the forward bond price equals the expected future bond price, the forward yield does not necessarily equal the expected future yield

Relationship Between Bond Yields and Prices (Figure 29.1, page 668)



Convexity Adjustment for Bond Yields

(Eqn 29.1, p. 668)

- Suppose a derivative provides a payoff at time T dependent on a bond yield, y_T observed at time T . Define:
- $G(y_T)$: price of the bond as a function of its yield
- y_0 : forward bond yield at time zero
- σ_y : forward yield volatility
- The expected bond price in a world that is FRN wrt $P(0, T)$ is the forward bond price
- The expected bond yield in a world that is FRN wrt $P(0, T)$ is
~~Forward Bond Yield~~
$$\frac{1}{2} y_0^2 \sigma_y^2 T \frac{G''(y_0)}{G(y_0)}$$

Convexity Adjustment for Swap Rate

The expected value of the swap rate for the period T to $T+\tau$ in a world that is FRN wrt $P(0, T)$ is

$$\text{Forward Swap Rate} = \frac{1}{2} y_0^2 \sigma_y^2 T \frac{G''(y_0)}{G'(y_0)}$$

where $G(y)$ defines the relationship between price and yield for a bond lasting between T and $T+\tau$ that pays a coupon equal to the forward swap rate

Example 29.1 (page 670)

- An instrument provides a payoff in 3 years equal to the 1-year zero-coupon rate multiplied by \$1000
- Volatility is 20%
- Yield curve is flat at 10% (with annual compounding)
- The convexity adjustment is 10.9 bps so that the value of the instrument is $101.09/1.1^3 = 75.95$

Example 29.2 (Page 670-671)

- An instrument provides a payoff in 3 years = to the 3-year swap rate multiplied by \$100
- Payments are made annually on the swap
- Volatility is 22%
- Yield curve is flat at 12% (with annual compounding)
- The convexity adjustment is 36 bps so that the value of the instrument is $12.36/1.12^3 = 8.80$

Timing Adjustments

(Equation 29.4, page 672)

The expected value of a variable, V , in a world that is FRN wrt $P(0, T^*)$ is the expected value of the variable in a world that is FRN wrt $P(0, T)$ multiplied by

$$\exp\left[-\frac{\rho_{VR}\sigma_V\sigma_R R_0(T^* - T)}{1 + R_0/m} T\right]$$

where R is the forward interest rate between T and T^* expressed with a compounding frequency of m , σ_R is the volatility of R , R_0 is the value of R today, σ_V is the volatility of F , and ρ is the correlation between R and V

Example 29.3 (page 672)

- A derivative provides a payoff 6 years equal to the value of a stock index in 5 years. The interest rate is 8% with annual compounding
- 1200 is the 5-year forward value of the stock index
- This is the expected value in a world that is FRN wrt $P(0,5)$
- To get the value in a world that is FRN wrt $P(0,6)$ we multiply by 1.00535
- The value of the derivative is $1200 \times 1.00535 / (1.08^6)$ or 760.26

Quantos

(Section 29.3, page 673)

- Quantos are derivatives where the payoff is defined using variables measured in one currency and paid in another currency
- Example: contract providing a payoff of $S_T - K$ dollars (\$) where S is the Nikkei stock index (a yen number)

Diff Swap

- Diff swaps are a type of quanto
- A floating rate is observed in one currency and applied to a principal in another currency

Quanto Adjustment (page 674)

- The expected value of a variable, V , in a world that is FRN wrt $P_X(0, T)$ is its expected value in a world that is FRN wrt $P_Y(0, T)$ multiplied by $\exp(\rho_{VW}\sigma_V\sigma_W T)$
- W is the forward exchange rate (units of Y per unit of X) and ρ_{VW} is the correlation between V and W .

Example 29.4 (page 674)

- Current value of Nikkei index is 15,000
- This gives one-year forward as 15,150.75
- Suppose the volatility of the Nikkei is 20%, the volatility of the dollar-yen exchange rate is 12% and the correlation between the two is 0.3
- The one-year forward value of the Nikkei for a contract settled in dollars is $15,150.75e^{0.3 \times 0.2 \times 0.12 \times 1}$ or 15,260.23

Quantos continued

When we move from the traditional risk neutral world in currency Y to the traditional risk neutral world in currency X, the growth rate of a variable V increases by

$$\rho\sigma_V\sigma_S$$

where σ_V is the volatility of V , σ_S is the volatility of the exchange rate (units of Y per unit of X) and ρ is the correlation between the two

$$\rho\sigma_V\sigma_S$$

Siegel's Paradox

An exchange rate S (units of currency Y per unit of currency X) follows the risk-neutral process

$$dS = [r_Y - r_X] S dt + \sigma_S S dz$$

This implies from Itô's lemma that

$$d(1/S) = [r_X - r_Y + \sigma_S^2] (1/S) dt - \sigma_S (1/S) dz$$

Given that the process for S has a drift rate of $r_Y - r_X$, we expect the process for $1/S$ to have a drift of $r_X - r_Y$.

Can you explain this?

When is a Convexity, Timing, or Quanto Adjustment Necessary

- A convexity or timing adjustment is necessary when interest rates are used in a nonstandard way for the purposes of defining a payoff
- No adjustment is necessary for a vanilla swap, a cap, or a swap option